

eTrust: Understanding Trust Evolution in an Online World

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ABSTRACT

Most existing research about online trust assumes static trust relations between users. As we are informed by social sciences, trust evolves as humans interact. Little work exists studying trust evolution in an online world. Researching online trust evolution faces unique challenges because more often than not, available data is from passive observation. In this paper, we leverage social science theories to develop a methodology that enables the study of online trust evolution. In particular, we propose a framework of evolution trust, eTrust, which exploits the dynamics of user preferences in the context of online product review. We present technical details about modeling trust evolution, and perform experiments to show how the exploitation of trust evolution can help improve the performance of online applications such as rating and trust prediction.

Categories and Subject Descriptors

H3.3 [Information Storage and Retrieval]: Information Search and Retrieval—*Information filtering*

General Terms

Algorithms, Design, Experimentation

Keywords

Multi-faceted Trust, Trust Evolution, User Preference

1. INTRODUCTION

The notion of online trust has attracted increasing attention from the computer science community in recent years [8, 22, 15]. Trust plays a central role in helping users overcome perceptions of risk and insecurity [6], especially for online users who seek advice from trusted sources to make decisions. The trustworthiness of the users is often tantamount to the reliability of the information they provide. Trust is

widely exploited to help online users collect reliable information in applications such as high-quality reviews detection [19] and product recommendation [10].

Most existing work on online trust assumes static trust relationships between users [27, 8, 7]. However, trust evolves as humans interact based on the findings from social sciences [11, 30, 12]. Sociologists investigate the evolution of trust in a physical world [30, 29]. Recent years witness many trust related online applications such as intelligent recommender systems [21, 7], collaborative filtering [10, 2], review quality prediction [19] and viral marketing [26], but little work is on online trust evolution due to some unique challenges. For example, in an online world, most of the information that a sociologist collects for trust evolution may not be available. In a physical world, sociologists first invite a group of participants, usually a small group, and then record their sociometric information on interpersonal trust and conditions or situations for the change [30]. In an online world, users are distributed all over the world. Even if one could find a study group, it would be difficult to gather sociometrics on trust. In other worlds, passive observation is the modus operandi for studying trust evolution. Since it is important to study trust evolution, we ask whether we can study online trust evolution and how we can gather online information to model trust evolution.

An important characteristic of product review sites such as Epinions¹ is that there exist trust networks among users. Such sites provide a sensible platform to study trust in an online world [8, 24, 18, 20]. Figure 1 shows a simple online rating system from Epinions at two different time points, denoted as T_1 and T_2 . There are two types of objects: users (u_1 to u_5) and items (I_1 to I_5), and two types of actions: establishing trust relations among users and creating ratings from users to items. The rating system evolves over time, highlighted in Figure 1(b), when (1) new users (e.g., u_5) and new items (e.g., I_5) are added; and (2) new trust relations (e.g., $u_2 \rightarrow u_1$) and new ratings (e.g., $u_5 \rightarrow I_4$) are created. The dynamic online rating system on product review sites serves an observable environment to investigate online trust evolution. First, trust networks with temporal information from product review sites are available, including time points when users entered and the trust relations established. Second, sources, reflecting the changing of user preferences, are available from product review sites. The dynamics of user preferences can be captured by their rating information, widely exploited for collaborative filtering [4, 14, 16]. The evolution of people's trust can be witnessed

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¹<http://www.epinions.com>

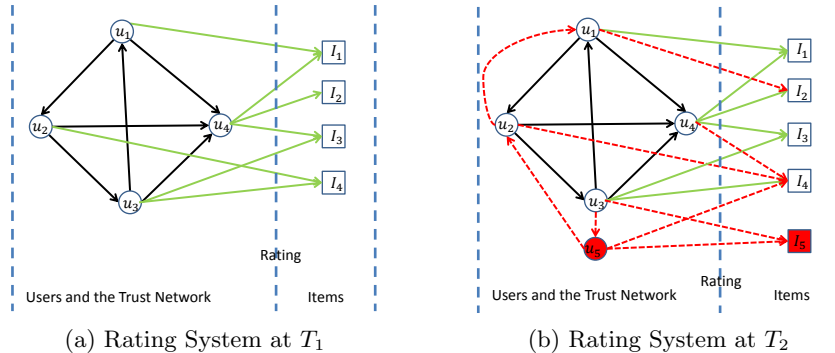


Figure 1: A Simple Example of Rating System at Time T_1 and T_2

via the changes of their preferences [11]. For example, when people are interested in “Electronics” at time t , they trust experts in “Electronics” while people shift their preferences to “Sports” at time $t+1$, they trust experts in this facet. Ziegler et al. [32] pointed out that there is a strong and significant correlation between trust and user preference similarity. The more similar two persons are, the greater the trust between them exists. In other words, trust relationships will evolve with the drifting of user preferences.

In this paper, we make a first attempt to study online trust evolution by exploiting the dynamics of user preferences in the context of online product review. Main contributions of this work include:

- Providing a methodology to study the evolution of trust in an online world;
- Proposing a framework, eTrust, to understand trust evolution by exploiting the dynamics of user preferences of online product review;
- Presenting findings from this eTrust study about the change of user preferences w.r.t. users or items, and trust relations; and
- Evaluating eTrust via online applications such as rating and trust prediction using real-world data.

The rest of paper is organized as follows. The problem of trust evolution is formally stated in Section 2. Section 3 introduces the details about our proposed framework, eTrust. Section 4 discusses applications of eTrust. Section 5 presents experimental results and findings about eTrust. Section 6 briefly reviews related work. Finally, Section 7 concludes this study with future work.

2. PROBLEM STATEMENT

A rating system of product review sites consists of two types of objects with two different actions, as shown in Figure 1(a). Let $\mathcal{U}_t = \{u_1, u_2, \dots, u_{n_t}\}$ be the set of users at time t (e.g., $\mathcal{U}_{T_1} = \{u_1, u_2, u_3, u_4\}$) where n_t is the number of users at time t . $\mathcal{I}_t = \{I_1, I_2, \dots, I_{m_t}\}$ is the set of items at time t (e.g., $\mathcal{I}_{T_1} = \{I_1, I_2, I_3, I_4\}$) where m_t is the number of items at time t . Note that new users or new items may be added ² thus $\mathcal{U}_t \subseteq \mathcal{U}_{t+1}$ and $\mathcal{I}_t \subseteq \mathcal{I}_{t+1}$. Users can perform two types of actions in the rating system: establishing new

²We do not consider the deletion of existing users or items in this work.

trust relations and creating new ratings. Let $\mathbf{X}_t \in \mathbb{R}^{n_t \times n_t}$ denote the trust network at time t where $\mathbf{X}_t(i, j) = 1$ if u_i is trusted by u_j at time t and zero otherwise. Let $\mathbf{R}_t \in \mathbb{R}^{n_t \times m_t}$ represent the ratings at time t and r_{ij}^t is used to represent the entity at i -th row and j -th column, i.e., the rating of I_j given by u_i at time t .

Users may have quite different preferences over items from different facets. We assume that there are K latent facets for items and a user has similar preferences over items of the same latent facet. Let $\mathbf{p}_i^t \in \mathbb{R}_+^K$ denote the u_i ’s preference vector at time t and each element of \mathbf{p}_i^t , $\mathbf{p}_i^t(k)$, represent the u_i ’s preference over the k -th facet at time t . Let $\mathbf{q}_j \in \mathbb{R}_+^K$ be the characteristic vector of the item I_j and $\mathbf{q}_j(k)$ denote the characteristic of I_j in the k -th latent facet. Let $\mathcal{W}_t \in \mathbb{R}^{n_t \times n_t \times K}$ denote the multi-faceted trust relations at time t and $w_{i_v k}^t$ is used to denote the strength of trust relation from u_i to u_v in the k -th facet at time t .

With above notations, the trust evolution problem with the dynamics of user preferences can be stated below.

Given T time slices, $\{\mathcal{U}_t\}_{t=1}^T$, $\{\mathcal{I}_t\}_{t=1}^T$, $\{\mathbf{X}_t\}_{t=1}^T$ and $\{\mathbf{R}_t\}_{t=1}^T$, understand the evolution of trust (eTrust), i.e., $\{\mathcal{W}_t\}_{t=1}^T$, by exploiting the changing of user preferences, $\{\{\mathbf{p}_i^t\}_{u_i \in \mathcal{U}_t}\}_{t=1}^T$, which is formally defined as,

$$\text{eTrust} : \{\mathcal{U}_t, \mathcal{I}_t, \mathbf{X}_t, \mathbf{R}_t\}_{t=1}^T \rightarrow \{\{\mathbf{p}_i^t\}_{u_i \in \mathcal{U}_t}\}_{t=1}^T, \{\mathcal{W}_t\}_{t=1}^T \quad (1)$$

3. A TRUST EVOLUTION FRAMEWORK

One challenge to studying online trust evolution is lack of conventional sociological data, and passive observation is a common approach to data collection. The dynamic rating system on product review sites provides a new means to the study of online trust evolution. In the next subsections, we introduce a framework of trust evolution, eTrust, with details on modeling and parameter estimation.

3.1 eTrust - Modeling Trust Evolution

It is observed [32] that trust has a strong correlation with user preference similarity in rating systems, reflected in their rating information. Thus most existing work about trust is studied in the context of rating prediction [2, 27]. In this work, we explore the dynamics of user preferences to model trust evolution for rating prediction. The user-item pairs, (i, j) , and their known ratings at time t , r_{ij}^t , are stored in $\mathcal{O}_t = \{(i, j) | r_{ij}^t \text{ is known}\}$. We use \hat{r}_{ij}^t to distinguish the predicted rating from the known rating r_{ij}^t . A baseline model for rating prediction is the latent factor model [13] based on

user preferences \mathbf{p}_i^t and item characteristics \mathbf{q}_j ,

$$\hat{r}_{ij}^t = \mathbf{q}_j^\top \mathbf{p}_i^t = \sum_{k=1}^K \mathbf{q}_j(k) \mathbf{p}_i^t(k), \quad (2)$$

Another approach for rating prediction is based on the trust network, a variant of the nearest-neighborhood model [13] by considering multi-faceted trust relationships between users,

$$\hat{r}_{ij}^t = \frac{\sum_{v \in N_i^t} \sum_{k=1}^K w_{ivk}^t \mathbf{q}_j(k) r_{vj}}{\sum_{v \in N_i^t} \sum_{k=1}^K w_{ivk}^t \mathbf{q}_j(k)} \quad (3)$$

where N_i^t represents the set of people trusted by u_i at time t . Assume that $r_{vj}^{t_{vj}}$ ($t_{vj} < t$) is the rating given by u_v to I_j at time t_{vj} . The influence of u_v to u_i on the rating given to I_j at time t , or the impact of $r_{vj}^{t_{vj}}$ on r_{ij}^t , is related to the distance between t_{vj} and t . The earlier ratings reflect users' previous preferences and should have less influence on the current ratings [4]. The closer t_{vj} and t , the more strongly $r_{vj}^{t_{vj}}$ influences r_{ij}^t . Thus we choose an exponential time function to allow the influence of past ratings to decay gradually, which is widely adopted in such applications [4, 14]. Thus r_{vj} in Eq. (3) can be stated as,

$$r_{vj} = e^{-\eta_i(t-t_{vj})} r_{vj}^{t_{vj}}, \quad (4)$$

$\eta_i \geq 0$ controls the user specific decay rate for u_i and should be learnt from the data. w_{ivk}^t in Eq. (3) is the trust strength between u_i and u_v in the k -th facet at time t . There is a strong and significant correlation between trust and user preference similarity [32]. Thus we define $\mathbf{s}_{ivk}^t \in \mathbb{R}^L$ as the preference similarity vector between u_i and u_v in the k -th facet at time t , based on their preferences in k -th facet at time t , i.e., $\mathbf{p}_i^t(k)$ and $\mathbf{p}_v^t(k)$, where L is the number of metrics to measure their preference similarity. We further assume a linear relation between trust strength w_{ivk}^t and \mathbf{s}_{ivk}^t [31], formally stated as,

$$w_{ivk}^t = f(\mathbf{w}^\top \mathbf{s}_{ivk}^t + b_i), \quad (5)$$

where $f: \mathbb{R} \rightarrow [0, 1]$ is an active function to control the trust strength, $0 \leq w_{ivk} \leq 1$ and b_i is a user specific bias of u_i .

The latent factor model does not incorporate the influence from the trust network while the neighborhood model does not consider the user preferences and item characteristics. Earlier work [13, 2, 27] indicated that a proper combination of these two models would help rating prediction. Therefore, we estimate the rating of u_i about I_j at time t as,

$$\hat{r}_{ij}^t = \alpha \sum_{k=1}^K \mathbf{q}_j(k) \mathbf{p}_i^t(k) + (1 - \alpha) \frac{\sum_{v \in N_i^t} \sum_{k=1}^K w_{ivk}^t \mathbf{q}_j(k) r_{vj}}{\sum_{v \in N_i^t} \sum_{k=1}^K w_{ivk}^t \mathbf{q}_j(k)} \quad (6)$$

In this formulation, the rating of u_i on I_j at time t is determined by two factors. One captures the preferences of u_i at time t and the characteristics of I_j , and the other is about user's trusted people. \hat{r}_{ij}^t should be influenced by u_i 's trusted people. w_{ivk}^t indicates their trust strength in the k -th facet. The stronger u_i trusts u_v , the more similar \hat{r}_{ij}^t is to r_{vj} . The second part models the influence from their trusted people. The parameter $\alpha \in [0, 1]$ adjusts the contributions from these two parts. Then the trust evolution problem, embedded in rating prediction, can be formulated

as the following minimization problem,

$$\begin{aligned} \min_{\mathbf{p}_i^t \geq 0, \mathbf{q}_j \geq 0, \eta_i \geq 0} & \sum_{t=1}^T \sum_{(i,j) \in \mathcal{O}_t} \left(r_{ij}^t - \alpha \sum_{k=1}^K \mathbf{q}_j(k) \mathbf{p}_i^t(k) - \right. \\ & \left. (1 - \alpha) \frac{\sum_{v \in N_i^t} \sum_{k=1}^K f(\mathbf{w}^\top \mathbf{s}_{ivk}^t + b_i) \mathbf{q}_j(k) r_{vj}}{\sum_{v \in N_i^t} \sum_{k=1}^K f(\mathbf{w}^\top \mathbf{s}_{ivk}^t + b_i) \mathbf{q}_j(k)} \right)^2 \\ & + \beta \left(\sum_{u_i \in \mathcal{U}_T} \sum_{t=t_{u_i}}^T \|\mathbf{p}_i^t\|_2^2 + \sum_{I_j \in \mathcal{I}_T} \|\mathbf{q}_j\|_2^2 \right. \\ & \left. + \|\mathbf{w}\|_2^2 + \sum_{u_i \in \mathcal{U}_T} \|b_i\|_2^2 + \sum_{u_i \in \mathcal{U}_T} \|\eta_i\|_2^2 \right) \\ & + \lambda \sum_{u_i \in \mathcal{U}_T} \sum_{t=t_{u_i}+1}^T \sum_{k=1}^K c(\mathbf{p}_i^t(k) - \mathbf{p}_i^{t-1}(k)) \end{aligned} \quad (7)$$

where the parameter β controls the quadratic regularized term to avoid overfitting, $c(\cdot)$ a function to model how user preferences change, and λ for the speed of change. When $\lambda \rightarrow 0$, we do not consider any relations of user preferences at different time points. When $\lambda \rightarrow +\infty$, the user preference vectors at different time points are restricted to be the same.

3.2 Estimating Parameters for eTrust

eTrust requires the following parameters: \mathbf{w} , b_i , η_i , the characteristics vector for each item I_j , \mathbf{q}_j , the user preference vector for each user u_i at each time slice t , \mathbf{p}_i^t . Different users enter the rating system at different time points, t_{u_i} for user u_i . Thus, for each u_i , we need to estimate his preferences from t_{u_i} to T , $\{\mathbf{p}_i^t\}_{t=t_{u_i}}^T$.

In our implementation, we use a projected gradient method to solve Eq. (7) and parameters can be updated as,

$$\begin{aligned} \mathbf{w} & \leftarrow \mathbf{w} - \gamma_{\mathbf{w}} \nabla_{\mathbf{w}}, \\ \forall u_i & \in \mathcal{U}_T \\ b_i & \leftarrow b_i - \gamma_{b_i} \nabla_{b_i}, \\ \forall u_i & \in \mathcal{U}_T \\ \eta_i & \leftarrow \max(0, \eta_i - \gamma_{\eta_i} \nabla_{\eta_i}) \\ \forall I_j & \in \mathcal{I}_T, k \in [1, K] \\ \mathbf{q}_j(k) & \leftarrow \max(0, \mathbf{q}_j(k) - \gamma_{\mathbf{q}_j(k)} \nabla_{\mathbf{q}_j(k)}), \\ \forall u_i & \in \mathcal{U}_T, t \in [t_{u_i}, T], k \in [1, K] \\ \mathbf{p}_i^t(k) & \leftarrow \max(0, \mathbf{p}_i^t(k) - \gamma_{\mathbf{p}_i^t(k)} \nabla_{\mathbf{p}_i^t(k)}), \end{aligned}$$

where $\gamma_{\mathbf{w}}$, γ_{b_i} , γ_{η_i} , $\gamma_{\mathbf{q}_j(k)}$ and $\gamma_{\mathbf{p}_i^t(k)}$ are learning step sizes, which are chosen to satisfy Goldstein Conditions [25]. $\nabla_{\mathbf{w}}$, ∇_{b_i} , ∇_{η_i} , $\nabla_{\mathbf{q}_j(k)}$ and $\nabla_{\mathbf{p}_i^t(k)}$ are the partial derivatives of the objective function in Eq. (7) with respect to \mathbf{w} , b_i , η_i , $\mathbf{q}_j(k)$ and $\mathbf{p}_i^t(k)$, respectively,

$$\begin{aligned} \nabla_{\mathbf{w}} & = 2\beta \mathbf{w} - 2(1 - \alpha) \sum_{t=1}^T \sum_{(i,j) \in \mathcal{O}_t} E_{ij}^t \frac{1}{(Q_{ij}^t)^2} \\ & \left(Q_{ij}^t \sum_{v \in N_i^t} \sum_{k=1}^K f'_{\mathbf{w}} \mathbf{q}_j(k) r_{vj} - P_{ij}^t \sum_{v \in N_i^t} \sum_{k=1}^K f'_{\mathbf{w}} \mathbf{q}_j(k) \right), \\ \nabla_{b_i} & = 2\beta b_i - 2(1 - \alpha) \sum_{t=1}^T \sum_{(i,j) \in \mathcal{O}_t} E_{ij}^t \frac{1}{(Q_{ij}^t)^2} \end{aligned}$$

$$\begin{aligned}
& \left(Q_{ij}^t \sum_{v \in N_i^t} \sum_{k=1}^K f'_{b_i} \mathbf{q}_j(k) r_{vj} - P_{ij}^t \sum_{v \in N_i^t} \sum_{k=1}^K f'_{b_i} \mathbf{q}_j(k) \right) \\
\nabla_{\eta_i} &= 2\beta\eta_i + 2(1-\alpha) \sum_{t=1}^T \sum_{(i,j) \in \mathcal{O}_t} \frac{E_{ij}^t}{Q_{ij}^t} \sum_{v \in N_i^t} \sum_{k=1}^K f \mathbf{q}_j(k) r_{vj} (t - t_{vj}) \\
\nabla_{\mathbf{q}_j(k)} &= 2\beta \mathbf{q}_j(k) - 2 \sum_{t=1}^T \sum_{(i,j) \in \mathcal{O}_t} E_{ij}^t \left(\alpha \mathbf{p}_i^t(k) + (1-\alpha) \frac{1}{(Q_{ij}^t)^2} \right. \\
& \left. (Q_{ij}^t \sum_{v \in N_i^t} f r_{vj} - P_{ij}^t \sum_{v \in N_i^t} f) \right), \\
\nabla_{\mathbf{p}_i^t(k)} &= 2\beta \mathbf{p}_i^t(k) - 2 \sum_{(i,j) \in \mathcal{O}_t} E_{ij}^t \left(\alpha \mathbf{q}_j(k) + (1-\alpha) \right. \\
& \left. \frac{1}{(Q_{ij}^t)^2} (Q_{ij}^t \sum_{v \in N_i^t} f'_{\mathbf{p}_i^t(k)} \mathbf{q}_j(k) r_{vj} - P_{ij}^t \sum_{v \in N_i^t} f'_{\mathbf{p}_i^t(k)} \mathbf{q}_j(k)) \right. \\
& \left. - 2(1-\alpha) \sum_{z \in T_i^t} \sum_{(z,j) \in \mathcal{O}_t} \frac{1}{(Q_{zj}^t)^2} (Q_{zj}^t f'_{\mathbf{p}_i^t(k)} \mathbf{q}_j(k) r_{ij} \right. \\
& \left. - P_{zj}^t f'_{\mathbf{p}_i^t(k)} \mathbf{q}_j(k)) \right) + H_i^t \tag{8}
\end{aligned}$$

where $f'_{\mathbf{w}}$, f'_{b_i} and $f'_{\mathbf{p}_i^t(k)}$ are the partial derivatives of f with respect to \mathbf{w} , b_i and $\mathbf{p}_i^t(k)$, respectively. T_i^t denotes the users who trust u_i at time t . E_{ij}^t , P_{ij}^t , Q_{ij}^t and H_i^t are defined as,

$$\begin{aligned}
E_{ij}^t &= r_{ij}^t - \alpha \sum_{k=1}^K \mathbf{q}_j(k) \mathbf{p}_i^t(k) - \\
& (1-\alpha) \frac{\sum_{v \in N_i^t} \sum_{k=1}^K f(\mathbf{w}^\top \mathbf{s}_{ivk}^t + b_i) \mathbf{q}_j(k) r_{vj}}{\sum_{v \in N_i^t} \sum_{k=1}^K f(\mathbf{w}^\top \mathbf{s}_{ivk}^t + b_i) \mathbf{q}_j(k)}, \\
P_{ij}^t &= \sum_{v \in N_i^t} \sum_{k=1}^K f(\mathbf{w}^\top \mathbf{s}_{ivk}^t + b_i) \mathbf{q}_j(k) r_{vj}, \\
Q_{ij}^t &= \sum_{v \in N_i^t} \sum_{k=1}^K f(\mathbf{w}^\top \mathbf{s}_{ivk}^t + b_i) \mathbf{q}_j(k), \\
H_i^t &= \begin{cases} -\lambda c'_{\mathbf{p}_i^t(k)}(\mathbf{p}_i^{t+1}(k) - \mathbf{p}_i^t(k)), & t = t_{u_i} \\ \lambda c'_{\mathbf{p}_i^t(k)}(\mathbf{p}_i^t(k) - \mathbf{p}_i^{t-1}(k)), & t = T \\ \lambda (c'_{\mathbf{p}_i^t(k)}(\mathbf{p}_i^t(k) - \mathbf{p}_i^{t-1}(k)) - c'_{\mathbf{p}_i^t(k)}(\mathbf{p}_i^{t+1}(k) - \mathbf{p}_i^t(k))), & t_{u_i} < t < T \end{cases}
\end{aligned}$$

Next we discuss our choice of $f(\cdot)$ and $c(\cdot)$ for eTrust. $f(\cdot)$ is an active function to control the estimated trust strength in $[0, 1]$. It should be real-valued and differentiable function and the range of this function is limited in $[0, 1]$. However, we still lack the detailed description about this function. In this case, a sigmoid function is often used [1]. It exhibits a progression from small beginnings that accelerates and approaches a climax over the domain, formulated as,

$$f(\mathbf{w}^\top \mathbf{s}_{ivk}^t + b_i) = \frac{1}{1 + \exp(-(\mathbf{w}^\top \mathbf{s}_{ivk}^t + b_i))}. \tag{9}$$

Then $f'_{\mathbf{w}}$, f'_{b_i} and $f'_{\mathbf{p}_i^t(k)}$ are formally stated as,

$$\begin{aligned}
f'_{\mathbf{w}} &= f(1-f) \mathbf{s}_{ivk}^t, \\
f'_{b_i} &= f(1-f), \\
f'_{\mathbf{p}_i^t(k)} &= f(1-f) \mathbf{w}^\top (\mathbf{s}_{ivk}^t)'_{\mathbf{p}_i^t(k)}. \tag{10}
\end{aligned}$$

$c(\cdot)$ is used to capture how user preferences change over time. User preferences are usually assumed to change smoothly over time [16]. Under this assumption, $c(\cdot)$, $c'_{\mathbf{p}_i^{t-1}(k)}$ and $c'_{\mathbf{p}_i^t(k)}$ are defined as follows:

$$\begin{aligned}
c(\mathbf{p}_i^t(k) - \mathbf{p}_i^{t-1}(k)) &= \|\mathbf{p}_i^t(k) - \mathbf{p}_i^{t-1}(k)\|_2^2, \\
c'_{\mathbf{p}_i^{t-1}(k)} &= -2(\mathbf{p}_i^t(k) - \mathbf{p}_i^{t-1}(k)), \\
c'_{\mathbf{p}_i^t(k)} &= 2(\mathbf{p}_i^t(k) - \mathbf{p}_i^{t-1}(k)). \tag{11}
\end{aligned}$$

4. APPLICATIONS OF eTrust

We discuss the role of eTrust in product review applications to help improve the performance of rating and trust prediction.

4.1 Rating Prediction

Rating prediction is the task of predicting a given user's ratings for a given item based on past ratings or other information, regarded as one of the most important algorithms for recommendation system. People tend to seek advices from their trusted friends thus trust networks are widely exploited to improve the task of rating prediction. Most existing trust network based methods assume single and static trust relations. However, rating systems in product review sites are highly dynamic and user preferences are drifting over time, indicating the evolution of trust relations. Thus eTrust, modeling the trust evolution, can be used to further improve the performance of prediction.

Given the previous ratings $\{\mathcal{O}_t\}_{t=1}^T$, the task of trust prediction aims to predict the ratings at time $T+1$, \mathcal{O}_{T+1} . The rating from u_i to I_j at time $T+1$ can be predicted as,

$$\begin{aligned}
\hat{r}_{ij}^{T+1} &= \alpha \mathbf{q}_j^\top \mathbf{p}_i^{T+1} \\
&+ (1-\alpha) \frac{\sum_{v \in N_i^{T+1}} \sum_{k=1}^K f(\mathbf{w}^\top \mathbf{s}_{ivk}^{T+1} + b_i) \mathbf{q}_j(k) r_{vj}}{\sum_{v \in N_i^{T+1}} \sum_{k=1}^K f(\mathbf{w}^\top \mathbf{s}_{ivk}^{T+1} + b_i) \mathbf{q}_j(k)}, \tag{12}
\end{aligned}$$

where \mathbf{w} is learnt from testing data. For the existing item, I_j , its characteristics vector, \mathbf{q}_j , is not evolved. For the existing user, u_i , b_i and η_i are independent on time slices, directly applied to $T+1$. We define the change speed of u_i 's preference in the k -facet, Z_{ik} , as,

$$Z_{ik} = \frac{1}{T - t_{u_i} - 1} \sum_{t=t_{u_i}+1}^T |\mathbf{p}_i^t(k) - \mathbf{p}_i^{t-1}(k)|, \tag{13}$$

then user preference at $T+1$, i.e., $\mathbf{p}_i^{T+1}(k)$, can be estimated from that at T by considering the change speed of the user preference as,

$$\mathbf{p}_i^{T+1}(k) = \mathbf{p}_i^T(k) + Z_{ik}. \tag{14}$$

At time $T+1$, new users and new items might be introduced; and a challenge for eTrust is how to address the cold-start problem where no historical ratings on items or users are available. Homophily [23] is employed to deal with the cold-start problem: similar users are more likely to trust each other. Therefore, for a new user, we first find his top- ℓ similar users based on their profiles to establish his trust network and then use the average user preferences from his trust network to estimate his preferences. A similar strategy is adopted for each new item: the characteristics of a new item are estimated from its top similar items.

4.2 Trust Prediction

Trust between users is widely exploited by search and recommendation systems. Inferring unknown trust between users attracts more and more attention recently [8, 18, 3]. Trust propagation model is a popular model to derive trust relationships based on known trust relationships [8]. However, users' trust relationships usually follow a power-law distribution and 80% of users are in the long tail. Thus in practical, there is not enough information to apply this technique. Furthermore, the propagation model is not directly applicable to the new users who have little information. Previous work suggests that models, combining rating similarity with trust networks, can achieve better performance than trust propagation model [18, 3]. eTrust can be applied to the task of trust prediction.

Given trust networks and ratings before time T , we want to recommend trust relationships at time $T + 1$. eTrust learns its parameters by solving the problem in Eq. (7) based on the trust networks and rating information before T . Then the trust strength between u_i and u_v in the k -th facet can be calculated by Eq. (5). For a new user, we use the average user preferences of his top- ℓ similar users in terms of profiles to represent his preferences. After obtaining trust strengths for each facet, the overall strength can be computed as,

$$w_{iv}^{T+1} = \frac{1}{\|\mathbf{p}_i^{T+1}\|_2} \sum_{k=1}^K \mathbf{p}_i^{T+1}(k) w_{ivk}^{T+1}, \quad (15)$$

5. EXPERIMENTS

In this section, we conduct experiments to evaluate the proposed framework, eTrust. We first introduce the details about the dataset used in the experiments; next describe the findings and results of applying eTrust to online applications, i.e., rating prediction and trust prediction.

5.1 Dataset and Experiment Setting

To study the evolution of trust, we collect a dataset from a popular product review site, Epinions, in the month of May, 2011. In Epinions, people can rate various products and add members to their trust networks or "Circle of Trust". We started with a set of most active users and then did breadth-first search until no new users could be found. For each user, we collected their profiles (user name, location, real name, registration time, self-description, and favorite websites), product rating entries and trust relations. For each product rating entry, we collected the time point of the entry, product name, category and the rating. For each trust relation, we collected trustee, trustor and the time point when the trust relation established.

We compute the trustors and trustees for each user and the distributions are shown in Figure 2(a) and Figure 2(b). Most people have few trustors and trustees, while a few users have an extremely high number of trustees or trustors: suggesting a power-law distribution that is typical in social networks. We check the distributions of new actions and new objects w.r.t. the current number of actions and objects. The results for new actions and new objects are shown in Figure 2(c) and Figure 2(d), respectively: also suggesting power-law-like distributions. Some other statistics of the dataset are shown in Table 1.

The earliest rating was published on Jul 05, 1999 and the latest one was on May 08, 2011. However, the temporal information about the trust relations established before

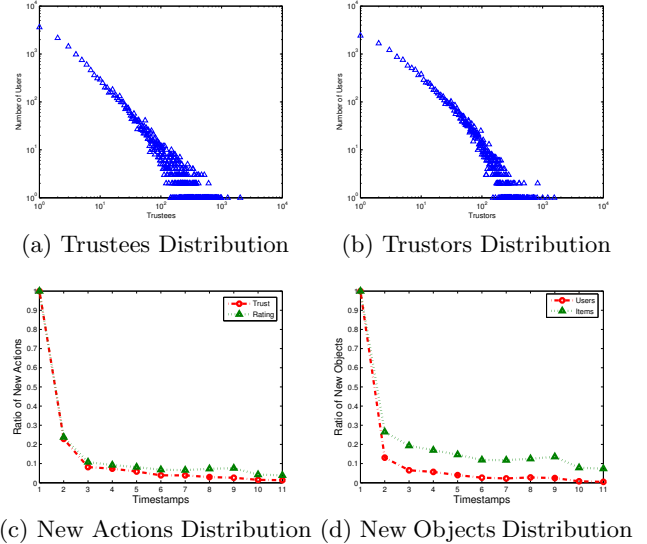


Figure 2: Distributions in Epinions

Table 1: Statistics of the Dataset

# of Users	22,166
# of Items	296,277
# of Categories	27
# of Ratings	922,267
# of Links	355,813
First Rating on	Jul 05 1999
Last Rating on	May 08 2011
Trust Network Density	0.0014
Clustering Coefficient	0.1518

11th January, 2001 is not available from Epinions. Therefore we split the whole dataset into 11 timestamps, i.e., $\mathcal{T} = \{T_1, \dots, T_{11}\}$, where T_1 covers the data before 11th January, 2001, T_{11} contains data after 11th January, 2010 and for T_2 to T_{10} , each of them contains one-year data. For example, T_2 contains data from 12th January, 2001 to 11th January, 2002. In the following experiments, we choose T_1 to T_{10} as the training set to estimate the parameters of eTrust.

In Eq. (7), \mathbf{s}_{ivk}^t is the preference similarity vector for u_i and u_v in the k -th facet at time t and in this work, we define the following 9 metrics of preference similarity as,

$$\begin{aligned} \mathbf{s}_{ijk}^t &= [\mathbf{p}_i^t(k), \mathbf{p}_j^t(k), \mathbf{p}_i^t(k) + \mathbf{p}_j^t(k), \mathbf{p}_i^t(k) - \mathbf{p}_j^t(k), \\ &\mathbf{p}_j^t(k) - \mathbf{p}_i^t(k), (\mathbf{p}_i^t(k) - \mathbf{p}_j^t(k))^2, \mathbf{p}_j^t(k)\mathbf{p}_i^t(k), \frac{\mathbf{p}_i^t(k)}{\mathbf{p}_j^t(k)}, \frac{\mathbf{p}_j^t(k)}{\mathbf{p}_i^t(k)}]. \end{aligned}$$

5.2 Findings about eTrust

Some interesting findings are summarized below.

- For each trust relation, x_{iv} , the evolution speed of this relation in the k -th facet (facet speed), f_{ivk} , is defined as,

$$f_{ivk} = \frac{1}{T - t_{iv} - 1} \sum_{t=t_{iv}+1}^T |w_{ivk}^t - w_{ivk}^{t-1}| \quad (16)$$

where t_{iv} is the time point when x_{iv} established and the overall evolution speed, F_{iv} , is defined as the sum of all K facet speed, $F_{iv} = \sum_{k=1}^K f_{ivk}$. We find that the evolution speed varies with facets and trust relations

within an open triad, shown as the left subgraph in Figure 3, are more likely to evolve than those within a closed triad, demonstrated in the right subgraph in Figure 3. On average, the evolution speed of trust relations within an open triad is 6.12 times of the speed of those within a closed triad.

- Users with similar preferences are more likely to trust each other, demonstrating the homophily [23] in the trust network. For example, the average preference similarity between trusted users is 3.44 times as that of users without trust relations at T_{10} and we have similar observations in other time slices.
- User preferences drift over time which can be observed in Figure 4, depicting user preferences from 2001 to 2009, i.e., $\{\{\mathbf{p}_i^t\}_{u_i \in \mathcal{U}_T}\}_{t=1}^9$. The change of user preference from t to $t+1$, Y_t^{t+1} , is defined as,

$$Y_t^{t+1} = \frac{1}{n_{t+1}} \sum_{u_i \in \mathcal{U}_{t+1}} \|\mathbf{p}_i^{t+1} - \mathbf{p}_i^t\| \quad (17)$$

we find that the change of user preferences varies in different years. For example, the changes of user preferences from 2003 to 2004 and from 2005 to 2006 are 2.96 and 1.79 times of the average, respectively. This might be related to two big events happened in Epinions: Epinions was acquired by Shopping.com³ in 2003 and in turn was acquired by Ebay⁴ in 2005⁵.

- The speed of change, defined in Eq. (13), varies with people and facets. Figure 5 shows the average speed of change from 2001 to 2009 w.r.t. different facets. Different people have different speeds of change while user preferences change much faster for some facets than others. For example, the average changing speed for the 13-th latent facet is almost 60 times of that for 2-th latent facet.
- People have multiple and heterogeneous trust relations with others. For each facet, people only trust a part of their networks strongly. For example, on average, people trust only 15.8% of their trust networks for a specific facet when the number of latent facets are 20 at T_{10} . We also observe heterogeneous trust relations including heterogeneous pairs of reciprocal trust relations, transitive relations and co-citation relations. These observations are very consistent with the findings of our previous multi-faceted trust research [27].

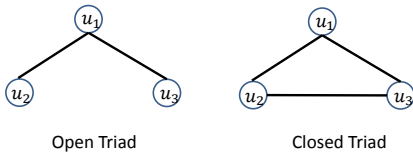


Figure 3: Open Triad vs Closed Triad

³<http://www.shopping.com/>

⁴<http://www.ebay.com/>

⁵<http://en.wikipedia.org/wiki/Epinions>

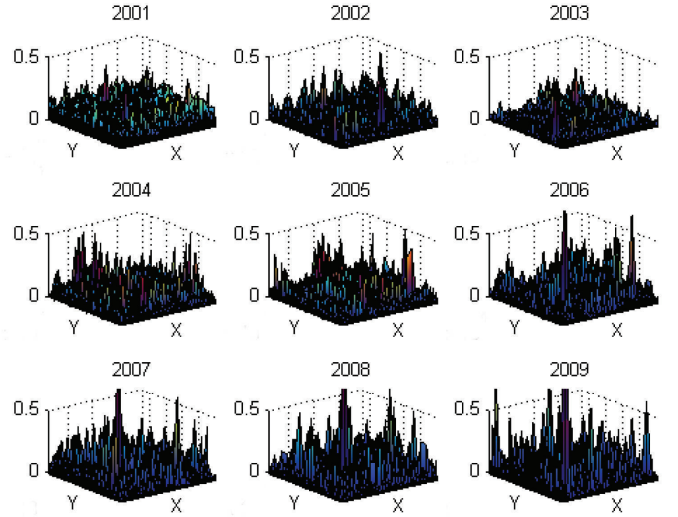


Figure 4: User Preference Evolution. Note that x-axis, y-axis and z-axis denote the facet ID, user ID and user preference, respectively.

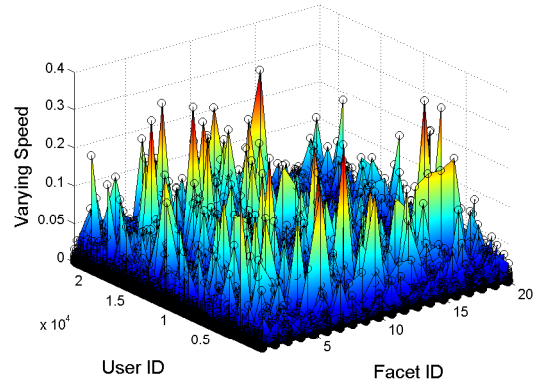


Figure 5: Speed of Changes w.r.t. Users and Facets

5.3 Rating Prediction

eTrust can be naturally applied to the online application of rating prediction, as discuss in Section 4.1. T_{11} is the testing set in this experiment and we further divide it into two parts: the ratings involved in new items or new users (cold-start problem), denoted as \mathcal{N} ; the remaining ratings from T_{11} denoted as \mathcal{K} . We closely examine T_{11} and find that \mathcal{N} contains 10.06% of the ratings at T_{11} .

We use Root Mean Squared Error (RMSE), a common metric, to evaluate prediction accuracy [13].

$$RMSE(\mathcal{M}) = \sqrt{\frac{\sum_{(i,j) \in \mathcal{M}} (r_{ij} - \hat{r}_{ij})^2}{|\mathcal{M}|}} \quad (18)$$

where $|\mathcal{M}|$ is the size of the testing set, \mathcal{M} . Note that eTrust can be easily applied to other rating prediction methods thus we aim to investigate whether considering dynamics of trust relations and user preferences allows us to improve the prediction performance, instead of comparing different rating prediction methods.

Table 2: RMSE of Rating Prediction in Epinions

	\mathcal{K}	\mathcal{N}	$\mathcal{K} + \mathcal{N}$
Mean	1.1054	1.1562	1.1106
NN	1.1092	1.1566	1.1148
MF	1.0804	1.1472	1.0872
MF+NN	1.0675	1.1392	1.0747
mTrust	1.0566	1.1375	1.0646
eTrust	1.0299	1.0783	1.0347

The prediction results are shown in the Table 2 and the methods listed in the table are defined as follows:

- *Mean*: the rating of an item is predicted by the mean of known ratings of the product;
- *MF*: this method is based on user preferences and item characteristics, however it assumes that user preferences do not evolve over time [13];
- *NN*: this model is a simple nearest-neighbor algorithm, assuming that a user has the similar rating as his trust network for the same item [13];
- *MF+NN*: this model is a combination of *NN* and *MF*, however, it considers heterogeneous strengths of trust relations [2];
- *mTrust*: this model considers the multi-faceted trust relations between users and assumes people place trust differently on different people [27];
- *eTrust*: our proposed method, considering the dynamics of both trust relations and user preferences.

Note that all baseline methods cannot tackle the cold-start problem thus we adopt the similar strategy as eTrust by considering their top- ℓ similar existing users (items) based on their profiles. For example, for *Mean*, the rating of a new item is predicted by the average rating of its top- ℓ similar items. The parameters of all methods are determined through cross-validation. For eTrust, the parameters are set as: $K = 20$, $\alpha = 0.3$, $\beta = 0.05$ and $\gamma = 0.1$.

We notice that RMSE values for all methods are very close to each other. However, small improvement in RMSE terms can have a significant impact on the quality of the top few recommendation [13]. As reported in [13], when the performance improved from 0.9025 to 0.8870 w.r.t. RMSE, it gains more than 50% relative improvement in terms of top few recommendation. *Mean* obtains better performance than *NN*. There are two main reasons. First, most of the time, the majority of users actual ratings are close to the average and by closely examining the dataset, we find that more than 70% users give a score of 4 or 5. Second, *NN* treats all trust relations equally, however, people may trust a part of their trust networks more strongly than others. When considering heterogeneous trust strengths, *MF+NN* outperforms both *MF* and *NN*, further demonstrating that people trust their friends differently while only give similar ratings with their strongly trusted friends.

NN and *MF+NN* assume single and homogeneous trust relations, however, people with multi-faceted interests and experts of different types suggest multiple and heterogeneous trust relations [27]. *mTrust* considers multi-faceted trust relations and obtains better performance than *MF+NN*. *eTrust*

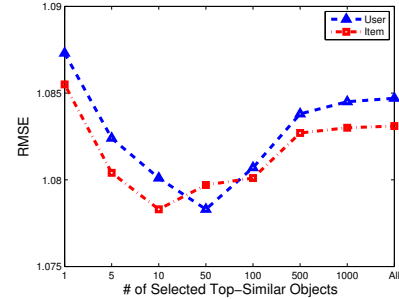


Figure 6: RMSE w.r.t. the Number of Selected Top Similar Users

consistently outperforms all other methods and we believe that this improvement is contributed by exploiting the dynamics of both trust relations and user preferences.

The performance of all methods degrades in \mathcal{N} . However, eTrust obtains much better performance than other methods, gaining 0.0592 absolute improvement in terms of RMSE. These results demonstrate that eTrust is more robust to the cold-start problem. We also investigate how the performance of eTrust varies with the number of selected top-similar users or items, ℓ , and the results are shown in Figure 6. The performance with $\ell = 1$ is always worst, even worse than that with all users or items. It suggests that users (items) with most similar profiles do not necessarily have most similar user preferences (item characteristics).

5.4 Trust Prediction

Trust plays an important role in helping collect reliable information for users in online communities. If trust relations can be predicted accurately, users can use these relations to make decisions on the reliability of information. For eTrust, trust relations can be inferred by user preferences with estimated \mathbf{w} and user specific bias $\{b_i\}$ by Eq. (5).

Data from T_{11} are treated as the testing set, separated into two parts: \mathcal{E} : trust relations established among existing users and \mathcal{N} : trust relations involved in new users, covering 23.51% of the whole trust relations at T_{11} . We follow the metric used in [17] to evaluate the performance of trust prediction. Let A be the set of pairs of users, having no relations at time T_{10} , and B denote the set of pairs of users, establishing trust relations at time T_{11} . Each trust predictor ranks pairs in A in decreasing order of confidence and we take the first $|B|$ pairs as the set of predicted trust relations, denoting as C . Then the prediction accuracy (PA) can be calculated as,

$$PA = \frac{|B \cap C|}{|B|} \quad (19)$$

where $|\cdot|$ denotes the size of a set.

The PA results are shown in Table 3 and the trust predictors mentioned in the table are defined as follows:

- *Simi*: it ranks the pairs according to their rating similarity (if available) and profile similarity, which only considers profile information or rating information.
- *TP*: the trust relations are inferred through trust propagation, only considering the existing trust network information [8].

Table 3: Accuracy of Trust Prediction in Epinions

	\mathcal{E} (%)	\mathcal{N} (%)	$\mathcal{E} + \mathcal{N}$ (%)
Simi	48.41	28.94	43.93
TP	45.47	N.A.	35.01
Simi+TP	50.19	28.94	45.31
eTrust	55.07	33.83	50.18

- *TP + Simi*: a combination of *Simi* and *TP*, integrating both the existing trust network and rating or profile similarity [3].
- *eTrust*: trust relations are predicted by eTrust through Eq. (5) and details are shown in the subsection 4.2.

Trust propagation always obtains the worst performance. *TP* predicts trust relationship between two users when there are existing trust relationships connecting them indirectly. However, connectivity information is not always enough for correct trust prediction, especially when trust connectivity is not high enough for applying propagation techniques. For example, *TP* cannot be applied to the dataset \mathcal{N} . *Simi* obtains better performance than *TP*, demonstrating that users with similar ratings or profiles are likely to trust each other. When considering both existing trust networks and similarity, *Simi+TP* outperforms both *TP* and *Simi*, indicating similarity information and existing trust networks are complementary with each other for trust prediction.

eTrust performs consistently best. For example, eTrust gains 9.72% and 14.45% relative improvement in \mathcal{E} and \mathcal{N} , respectively. There are two main reasons: (1) eTrust performs trust prediction facet by facet, overcoming the problem of the heterogeneous trust relations [27]; and (2) trust relations are inferred based on user current preferences. We observe that the performance of all predictors decreases in \mathcal{N} . However, based on profile similarity, *Simi* still obtains 28.94% in terms of prediction accuracy, much larger than a random predictor whose accuracy is less than 1%. This observation demonstrates the existence of homophily in trust networks: similar users are more likely to trust each other.

6. RELATED WORK

Trust has attracted more and more attention from computer science community [20, 5]. In recent years, many trust related online applications are proposed and we next briefly review some of them related to our work.

Rating Prediction. People are likely to seek advices from their trust networks thus trust networks are widely exploited in the task of rating prediction. In [20], several methods for incorporating trust networks are proposed to improve the performance of rating prediction. [22] studied and modeled the bidirectional effects between trust relations and product rating. Koren introduced some innovations to both latent factor models and neighborhood models. The factor and neighborhood models can now be smoothly merged, thereby building a more accurate combined model [13]. It reported that a proper combination of these two models can significantly improve the prediction performance. All these methods do not consider the changes of user purchase interest. However, customer preferences for products are drifting over time [14]. Ding et al. presented a novel algorithm to compute the time weights for different items by assigning a decreasing weight to old data [4]. Koren proposed a methodology and specific techniques for modeling the dy-

namics of user preferences in recommender systems [14]. It claimed that temporal dynamics in the data can have more significant impacts on accuracy than designing more complex learning algorithms.

Trust Prediction. Trust plays an important role in helping online people collect reliable information. If trust relations can be predicted accurately, users can use these relations to make decisions on the reliability of information. Guha et al. developed a formal framework of trust propagation schemes [8]. It first separates trust and distrust matrix and then performs operations on them to obtain the transitive trust between two nodes. The connection between trust and user similarity was studied in [32], and a strong and significant correlation is found between trust and similarity. The more similar two people are, the greater the trust between them is. In [3], rating similarity is exploited to enrich traditional trust propagation methods. This work demonstrated that predicting trust is more successful for pairs of users that are similar to each other if we combine the topology of the trust network with rating similarity.

Trust Strength Prediction is another direction of related research, which differs from trust prediction. The former focuses on modeling the strength of existed links rather than link existence. In [31] a latent variable model is developed to estimate relationship strength from various interaction activities and user similarities. In this model, relationship strength is modeled as the hidden effect of user similarities and it also impacts the nature and frequency of online interactions. Au et al. show heterogeneous trust strengths of trust relations in product review sites and a modified matrix factorization technique is proposed to estimate strengths of trust relations. Trust, as a social concept, naturally has multiple facets, indicating multiple and heterogeneous trust relations between users. Our previous work, mTrust, investigated multi-faceted trust relations between users. People place trust differently on different people and the work of mTrust demonstrates that trust strength can be inferred under the context of rating prediction [27].

7. CONCLUSIONS

In this paper, we study online trust evolution in the context of product review. By exploiting the correlation between user preferences and trust relations, we propose a framework, eTrust, to understand the evolution of trust in an online world and apply eTrust to various online applications such as rating prediction and trust prediction. Interesting findings are observed in our experiments using real-world data, Epinions; and eTrust can be applied to improve the performance of rating prediction and trust prediction. In our future work, we will continue our research on eTrust and investigate trust evolution in areas such as ranking evolution, useful message recommendation [9] and linked feature selection [28].

Acknowledgments

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